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# Maxwell's equations and Occam's razor

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Presentation of some key ideas from the 2nd edition of our book, titled "Maxwell-Dirac Theory and Occam's Razor: Unified Field, Elementary Particles, and Nuclear Interactions"

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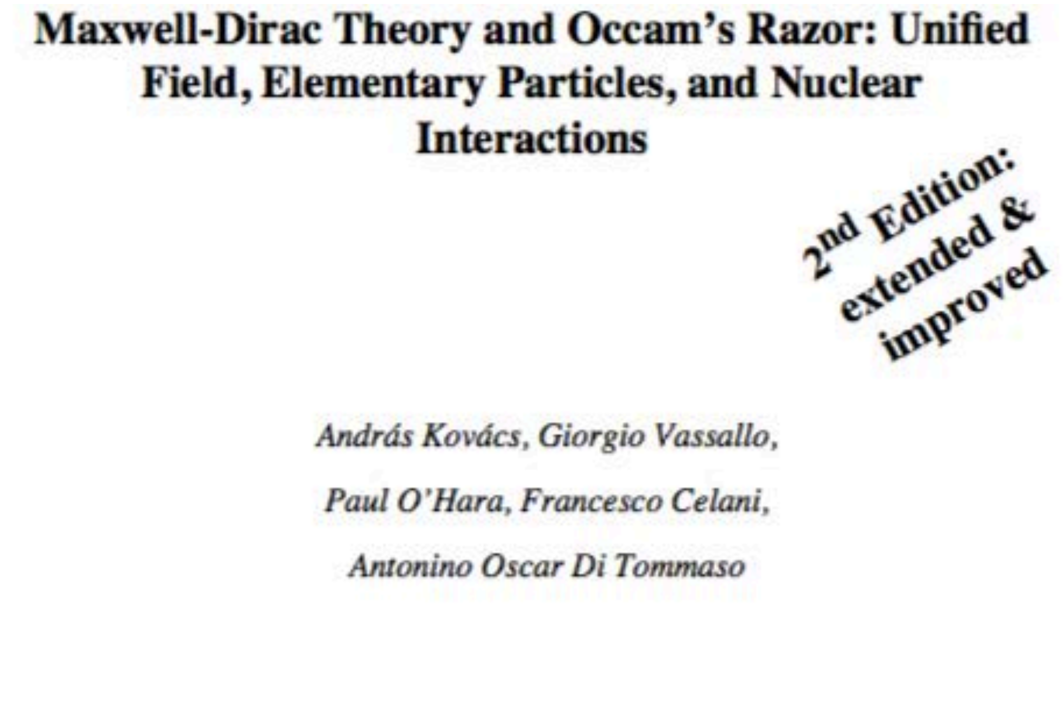
# Contents

# Methodology

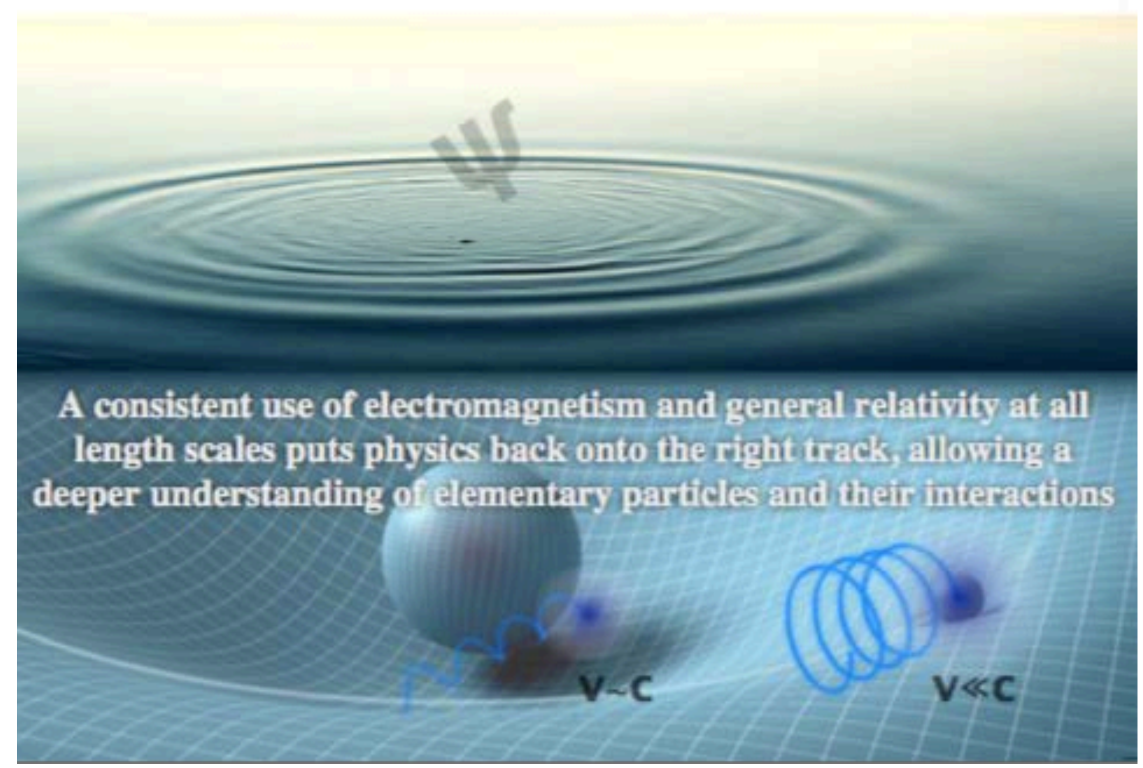
1. Occam's razor principle: **among different models that fit experimental data, the simplest one must be preferred**. One of the earliest formulation is by Ptolemy (2nd century): "We consider it a good principle to explain the phenomena by the simplest hypothesis possible".




2. **A consistent use of electromagnetism and general relativity at all length scales**.

Example:
electric energy density is given by $w_e = \epsilon_0 \frac{\boldsymbol{E}^2}{2}$

This expression is derived directly from Maxwell's equation, and we will consistently use it on ALL length scales.



# Brief overview of Clifford algebra

Clifford algebra is defined by the multiplication rule of its basis elements. The Clifford basis elements are defined to obey the following multiplication rules:

$$\mathbf{e}_t^2 = -1, \mathbf{e}_x^2 = \mathbf{e}_y^2 = \mathbf{e}_z^2 = 1$$

$$\mathbf{e}_i\mathbf{e}_j = -\mathbf{e}_j\mathbf{e}_i \; (i \neq j)$$

The $t$ index represents the time coordinate, and indices $x, y, z$ represent spatial coordinates. Thus the Clifford algebra basis elements can be identified with the unit vectors spanning our four dimensional space-time.

We use the following notation for bi-vectors: $\mathbf{e}_{ij} \equiv \mathbf{e}_i\mathbf{e}_j$. Similarly, tri-vectors are volume elements, and denoted as: $\mathbf{e}_{ijk} \equiv \mathbf{e}_i\mathbf{e}_j\mathbf{e}_k$. The dimensionality of an expression goes up through multiplication of different base vector components.

# Brief overview of Clifford algebra

A useful notation for a space-time vector is $Q = (Q_t\mathbf{e}_t, \mathbf{Q})$, where the bold capital notation is denoting a spatial vector, i.e. $\mathbf{Q} = q_x\mathbf{e}_x + q_y\mathbf{e}_y + q_z\mathbf{e}_z$. The product of two vectors is:

$$PQ = (P_t\mathbf{e}_t, \mathbf{P})\,(Q_t\mathbf{e}_t, \mathbf{Q}) = (-P_tQ_t + \mathbf{P}\cdot\mathbf{Q}) + (-P_t\mathbf{Q} + \mathbf{P}Q_t)\,\mathbf{e}_t + \mathbf{P}\times\mathbf{Q}\mathbf{e}_{xyz}$$

$$P \cdot Q \quad + \quad P \wedge Q$$

Scalar (symmetric) + Bi-vector (anti-symmetric)

We define the unitary pseudoscalar as $I \equiv \mathbf{e}_{txyz}$

We use the following definition of the operator $\boldsymbol{\partial}$ in space-time algebra

$$\boldsymbol{\partial} = \gamma_x\frac{\partial}{\partial x} + \gamma_y\frac{\partial}{\partial y} + \gamma_z\frac{\partial}{\partial z} + \gamma_t\frac{1}{c}\frac{\partial}{\partial t} = \nabla + \gamma_t\frac{1}{c}\frac{\partial}{\partial t}$$

# From vector potential to electromagnetic fields

Space-time derivative of vector potential yields electromagnetic fields:

$$\boldsymbol{\partial} \boldsymbol{A}_{\square} = \boldsymbol{\partial} \cdot \boldsymbol{A}_{\square} + \boldsymbol{\partial} \wedge \boldsymbol{A}_{\square} = S + \boldsymbol{F} = \boldsymbol{G}$$

<u>Traditional understanding:</u>

*S=0*

*(Lorenz gauge)*

$$\boldsymbol{F} = \frac{1}{c}\boldsymbol{E}\gamma_t + I\boldsymbol{B}\gamma_t$$

*(treated as a complex number)*

Why was it **assumed** that one may declare S to be zero? The philosophy of electromagnetism is based on the following hierarchy of entities:

**Charges** → Liénard–Wiechert potential formula → **Vector potential field** → Spacetime derivative → **Electromagnetic field**

At the output, theorists wanted the electromagnetic vector field, and this motivated to eliminate the scalar part.

The results seem to match the measured propagation of electromagnetic waves in space.

# What is wrong with the traditional perspective?


- Charges become external input objects to the theory of electromagnetism:
    - They inserted "by hand" into Maxwell's equation
    - It remains unclear "what charges are made of"
    - Consequently, charges are viewed as "point-like particles" (renormalization problem)

- It is not clear how charges couple to electromagnetic forces. Mainstream perspective is an outdated atomistic view:
    - Charges are thought to continuously exchange "force carrying photon particles" among each other (ping-pong game theory, dressed in fancy language)

- The "gauge theory" of electromagnetism leads to contradictions, and thus cannot be correct:

*G. Rousseaux "The gauge non-invariance of Classical Electromagnetism", arXiv:physics/0506203v1*

# Maxwell's equation and Occam's razor

**Vector potential field** → Spacetime derivative → **Electromagnetic fields and charges**

$$\boldsymbol{\partial} \boldsymbol{A}_{\square} = \boldsymbol{\partial} \cdot \boldsymbol{A}_{\square} + \boldsymbol{\partial} \wedge \boldsymbol{A}_{\square} = S + \boldsymbol{F} = \boldsymbol{G}$$

Simplified understanding:

*Charges* (← $S$) *Electromagnetic fields* (← $\boldsymbol{F}$)

Maxwell's equation is now written as:

$$\boldsymbol{\partial} \boldsymbol{G} = \boldsymbol{\partial}^2 \boldsymbol{A}_{\square} = 0$$

Charge is a type of electromagnetic field. → Electromagnetism can be formulated as a proper field theory.

# Charges move at the speed of light

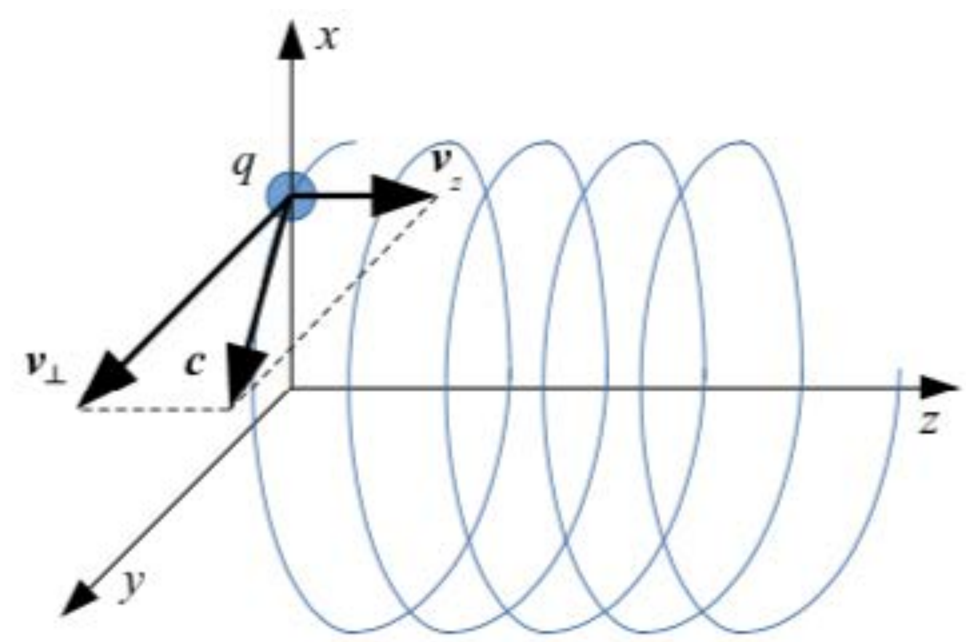
Expression for electric current:
$$\frac{1}{\mu_0}\boldsymbol{\partial} S = \frac{1}{\mu_0}\left(\gamma_x \frac{\partial S}{\partial x} + \gamma_y \frac{\partial S}{\partial y} + \gamma_z \frac{\partial S}{\partial z} + \gamma_t \frac{1}{c}\frac{\partial S}{\partial t}\right) = \boldsymbol{J}_{\Box e}$$

Charge conservation:
$$\frac{1}{\mu_0}\boldsymbol{\partial} \cdot (\boldsymbol{\partial} S) = \boldsymbol{\partial} \cdot \boldsymbol{J}_{\Box e} = \frac{\partial J_{ex}}{\partial x} + \frac{\partial J_{ey}}{\partial y} + \frac{\partial J_{ez}}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$

Combining the above two equations, we get a wave equation for charges:
$$\boldsymbol{\partial} \cdot (\boldsymbol{\partial} S) = \boldsymbol{\partial}^2 S = \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 S}{\partial t^2} = \nabla^2 S - \frac{1}{c^2}\frac{\partial^2 S}{\partial t^2} = 0$$

The above equation tells us that charges move at the speed of light!

This is the Zitterbewegung phenomenon:




Helical motion of an elementary charge $q$ moving at the speed of light, with $v_z^2 + v_\perp^2 = c^2$.

# History of the Zitterbewegung concept

De Broglie (1920s): to every elementary particle there is an associated oscillation angular frequency, which is $\omega_{DB}=mc^2/\hbar$.

Schrödinger (1930): Introduced the Zitterbewegung term, referring to a sinusoidal oscillation, at $\omega=2\omega_{DB}$ angular frequency. Amplitude is similar to reduced Compton wavelength.

Hönl (1938): Helicoidal electron Zitterbewegung model

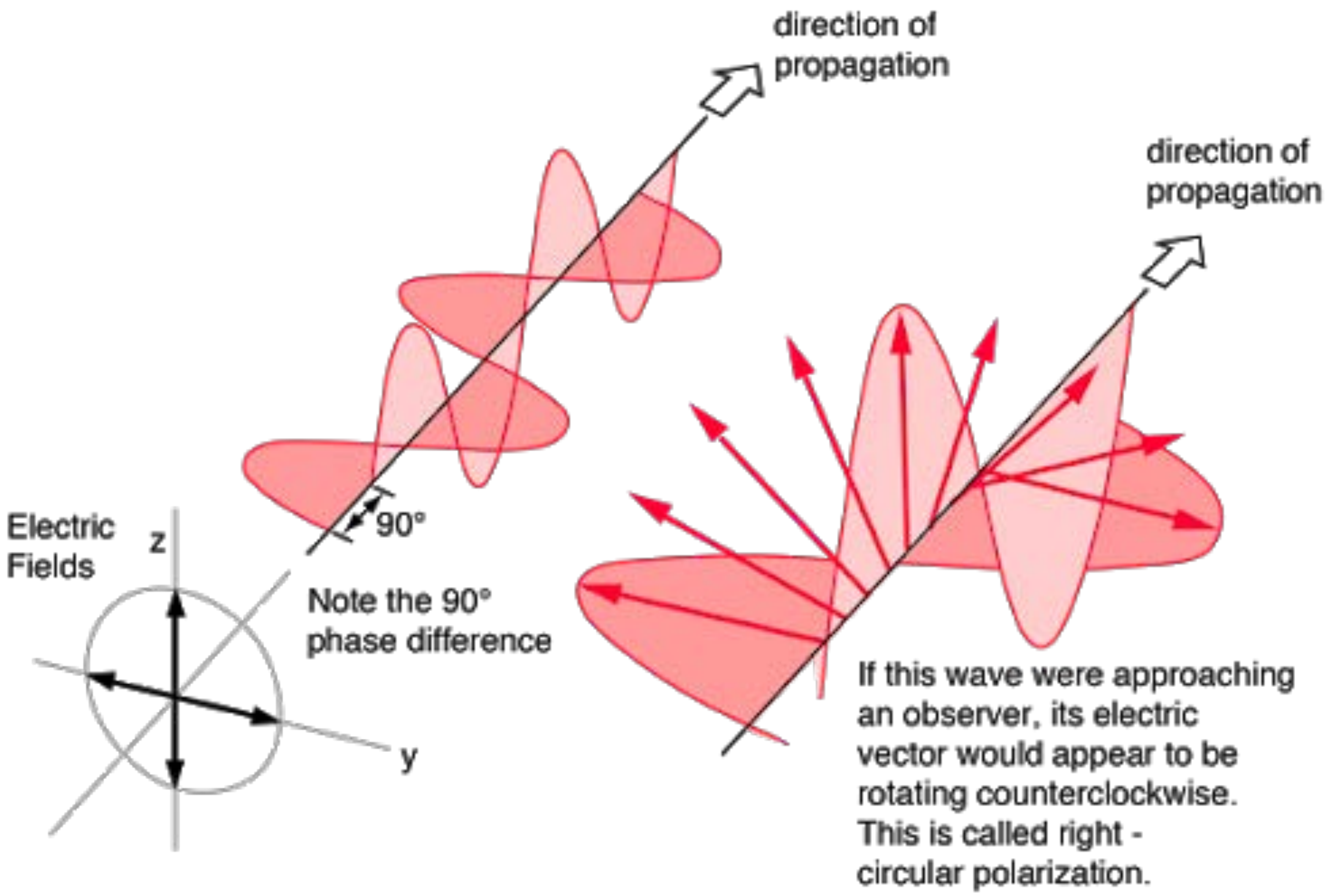
Kerson Huang (1952): More elaborate study of helicoidal Zitterbewegung, electron spin is derived from this helicoidal motion.
*This study was probably written by Huang's teacher*

Marcel Riesz (1950s): applied Clifford algebra to study electromagnetism

David Hestenes (1970s): study of helicoidal Zitterbewegung with the help of Clifford algebra

Giorgio Vassallo (2017): derivation of helicoidal electron Zitterbewegung from Maxwell's equations. The associated angular frequency is $\omega_{DB}$.

# Electromagnetic induction in a transversal wave




Transversal waves: Electric and Magnetic fields induce each other.

Dynamics of mutual induction is given by Maxwell's equation.

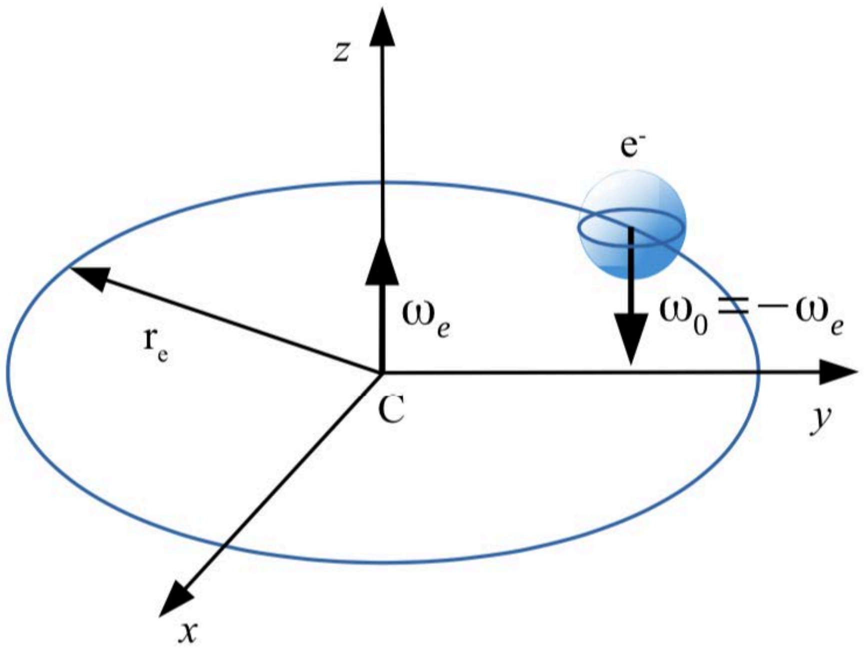
The electric and magnetic energy contents of the wave are equal: $W_m = W_e$.

Note 1: S=0 in a light wave (the Lorenz gauge can be applied here)

Note 2: Consider a circularly polarized wave, where the field is given by: $\boldsymbol{F} = \frac{1}{c}\boldsymbol{E}\gamma_t + I\boldsymbol{B}\gamma_t$

Upon multiplying by its "complex conjugate", we get $FF^* = B^2 - E^2$.
This quantity is Lorenz invariant.
That is not coincidence: it is in fact the electromagnetic Lagrangian.

# What is the electron size?




Electric and Magnetic fields induce each other also within the electron, but field topology is different.

Electric energy density: $w_e = \frac{1}{2}\epsilon_0 E^2 = \frac{\epsilon_0}{2}\left(\frac{1}{4\pi\epsilon_0}\cdot\frac{e}{r^2}\right)^2 = \frac{1}{32\pi^2\epsilon_0}\cdot\frac{e^2}{r^4}$

Total electric energy:

$$W_e = \frac{e^2}{32\pi^2\epsilon_0}\int_{r_0}^{\infty}\frac{1}{r^4}\cdot 4\pi r^2 dr = \frac{e^2}{8\pi\epsilon_0}\int_{r_0}^{\infty}\frac{1}{r^2}dr = -\frac{e^2}{8\pi\epsilon_0}\left.\frac{1}{r}\right|_{r_0}^{\infty} = \frac{e^2}{8\pi\epsilon_0 r_0}$$

With $r_0$=2.82 fm, $\mathbf{W_e}$**=255.5 keV**

} The physical meaning of charge radius

Total magnetic energy:

$$W_m = \frac{1}{2}\phi_e I_e = \frac{1}{2}\cdot 2\pi\frac{\hbar}{e}\cdot\frac{ec}{2\pi r_e} = \frac{\hbar c}{2r_e}$$

With $r_e$=386.16 fm, $\mathbf{W_m}$**=255.5 keV**

} The physical meaning of reduced Compton radius

**The electron mass is electromagnetic field energy.**

# Skeptic #1: what about accelerator experiments

Skeptic statement #1: high energy accelerator collisions show the electron to be very tiny.

Charges always move at *c*; thus ZBW radius shrinks when the particle moves in *z* direction:

$$v_z^2 + v_\perp^2 = c^2,$$

$$v_z^2 + \omega_e^2 r^2 = \omega_e^2 r_e^2 = c^2.$$

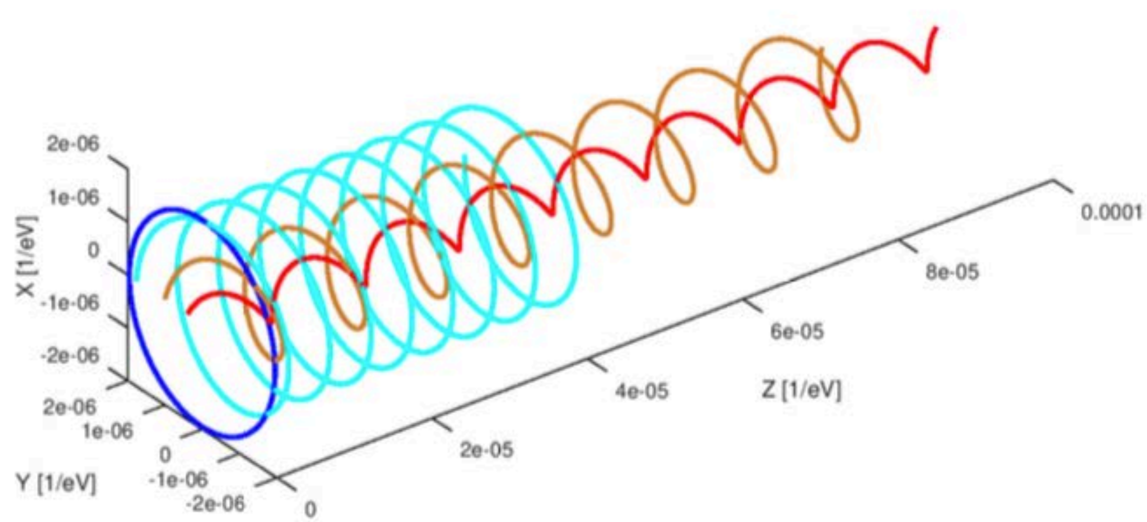
$$\frac{r^2}{r_e^2} = 1 - \frac{v_z^2}{c^2}$$

$$r = r_e \sqrt{1 - \frac{v_z^2}{c^2}}.$$

As shown in the previous slide, relativistic mass is inversely proportional to the ZBW radius:

$$m = \frac{m_e}{\sqrt{1 - \frac{v_z^2}{c^2}}}$$ ⟶ this is Einstein's formula

v/c = 0 , 0.43, 0.86, 0.98

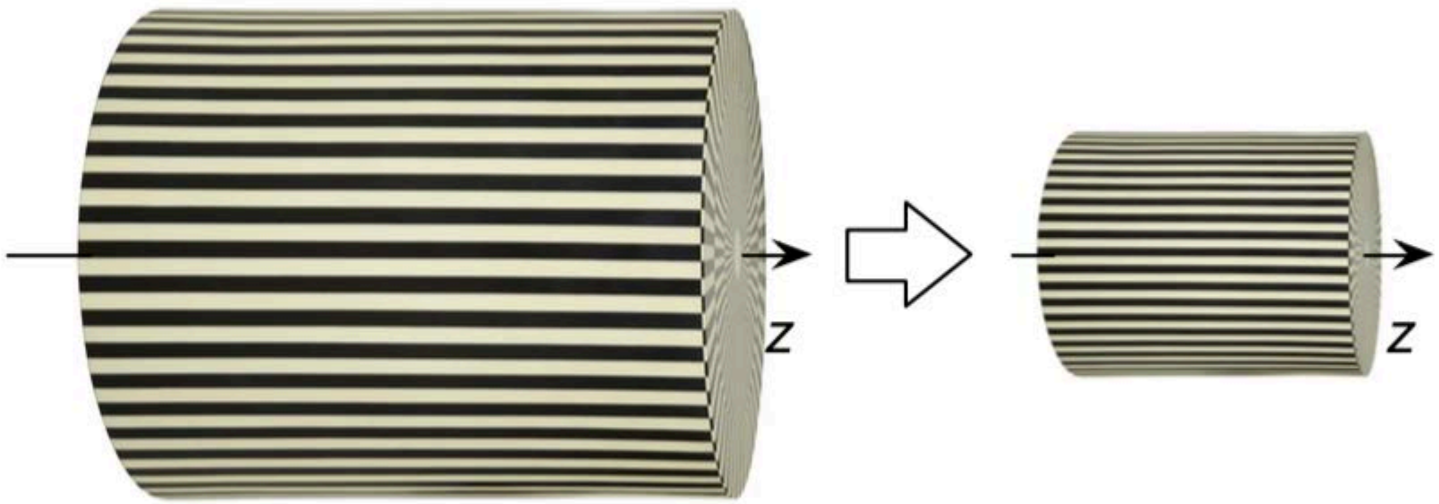



**The electron shrinks as its relativistic mass increases.**

# Skeptic #2: what about Lorentz contraction

Skeptic statement #2: Lorentz contraction shrinks the electron along the *z* axis only, the electron would appear as an ellipsoid from a moving frame.

One must not forget the Doppler effect, which applies to all electromagnetic waves.
Simple scenario: the electron moves along *z* axis at a certain speed, its position is unknown along the *z* axis due to Heisenberg uncertainty. It appears as a wave circulating in the *x*-*y* plane.




Lorentz contraction shrinks the electron by $\gamma_L$ along the *z* axis.

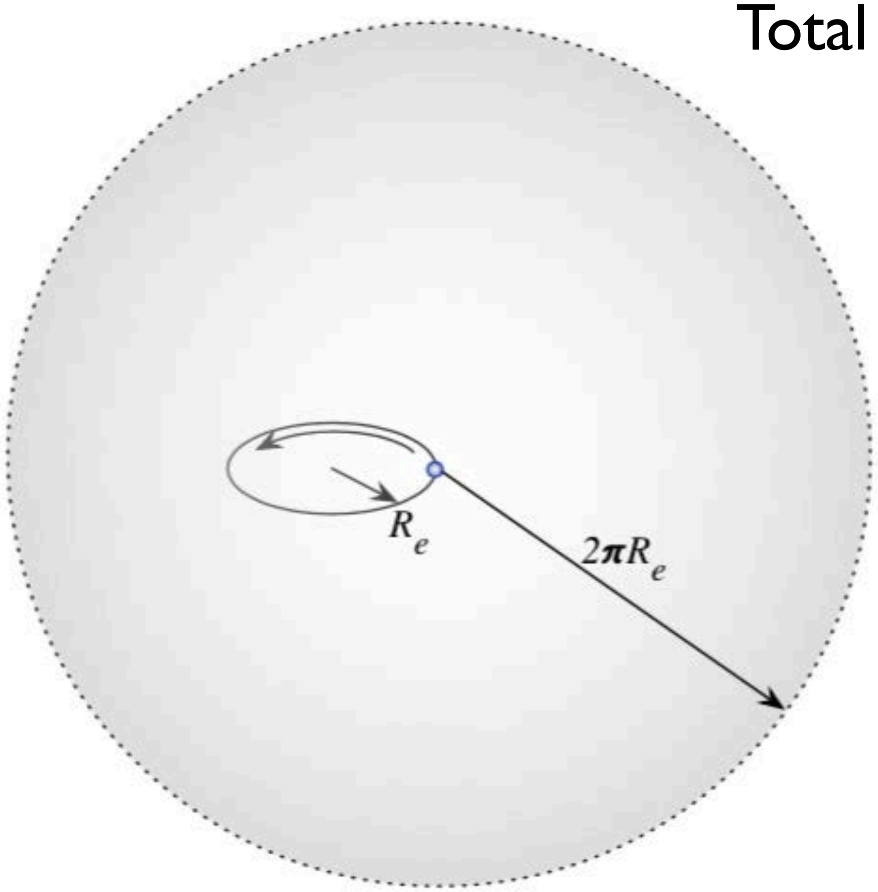
The transversal Doppler effect shrinks the electron by $\gamma_L$ along the *x*-*y* axes.

**The electron always retains a spherical charge surface.**

# Anomalous magnetic moment

The magnetic moment is traditionally given by the $\mu = \frac{e\hbar}{2m_e}$ formula

The only non-constant factor is the electron mass. Since mass is derived from E-M induction, excess magnetic moment indicates incomplete induction.




Total electric energy:

$$W_e = \frac{e^2}{32\pi^2\epsilon_0}\int_{r_0}^{\infty}\frac{1}{r^4}\cdot 4\pi r^2 dr = \frac{e^2}{8\pi\epsilon_0}\int_{r_0}^{\infty}\frac{1}{r^2}dr = -\frac{e^2}{8\pi\epsilon_0}\frac{1}{r}\bigg|_{r_0}^{\infty} = \frac{e^2}{8\pi\epsilon_0 r_0}$$

The meaning of g=1: complete E-M induction during a single ZBW circle.

Electric energy within $2\pi R_e$:

$$W_{e,2\pi R} = \frac{e^2}{8\pi\epsilon_0}\int_{r_0}^{2\pi R_e}\frac{1}{r^2}dr = -\frac{e^2}{8\pi\epsilon_0}\frac{1}{r}\bigg|_{r_0}^{2\pi R_e} = \frac{e^2}{8\pi\epsilon_0}\left(\frac{1}{r_0} - \frac{1}{2\pi R_e}\right)$$

Ratio of energies:

$$\frac{W_{e,2\pi R}}{W_e} = 1 - \frac{r_0}{2\pi R_e} \longrightarrow g = \left(1 - \frac{r_0}{2\pi R_e}\right)^{-1}$$

is the real g-factor.

# g-factor of the electron and proton

Electron: $r_0$=2.82 fm, $R_e$=386.16 fm

$$g = \left(1 - \frac{r_0}{2\pi R_e}\right)^{-1} \approx 1 + \frac{\alpha}{2\pi}$$ ⟶ this is Schwinger's formula

**Experimental: g = 1.00115965, Calculation: g = 1.00116276 (6 digits accuracy)**

Proton: for a long time, its radius value was uncertain (referred as the "proton radius puzzle")

In 2020, a much improved measurement accuracy was achieved*: $r_0$=0.8482 fm, R=0.2103 fm

$$g_p = \left(1 - \frac{r_p}{2\pi R_p}\right)^{-1}$$

**Experimental: g = 2.7929, Calculation: g = 2.7927 (99.99% accuracy)**

**A. Grinin et al "Two-photon frequency comb spectroscopy of atomic hydrogen", Science, Volume 370.6520 (2020)*

# Charge quantization

Maxwell's equation is linear in terms charge density and field strength:

$$\partial A_{\square} = \partial \cdot A_{\square} + \partial \wedge A_{\square} = S + F = G$$

How to explains then charge quantization?

Electric charges curve spacetime

↓

When spacetime curvature is taken into account, the effective wave equation is no longer linear.

↓

Giorgio Vassallo derives the geometric condition for charge quantization in the book.

↓

The next challenge is to mathematically derive the elementary charge quantum.

$$\left[\alpha^{-1} = e^{-2} \approx 137.035989\right]_{NU}$$

$e$

Magnetic flux quantum

$$\left[\Phi_M = 2\pi/e\right]_{NU}$$

# Electromagnetic symmetries

Lorentz boost: preserves $E^2 - B^2$ $\longrightarrow$ SU(2) symmetry group

Spatial rotation: preserves $S^2 + E^2 + B^2$ $\longrightarrow$ SU(2) symmetry group (double coverage)

There is another transformation* which preserves $S^2 + E^2 + B^2$ : it is $\gamma_t \to e^{I\theta}\gamma_t$

$\downarrow$

U(1) symmetry group

Note:

- $I$ is the Clifford pseudo-scalar,
- $\boldsymbol{\gamma}_t$ is the unit vector along the time axis
- $(S^2 + E^2 + B^2)/2$ is the energy density

**Similar approach: Gerrit Coddens "Rendering SU(3) intuitive: Symmetries of Lorentz tensors" (2018)*

# Electromagnetic symmetries

The symmetry group of $S^2+E^2+B^2$ preserving transformations is:
$U(1) \times SU(2)$

*Noether's theorem: every symmetry corresponds to a conserved physical quantity*

Isospin conservation

Spin (angular momentum) conservation

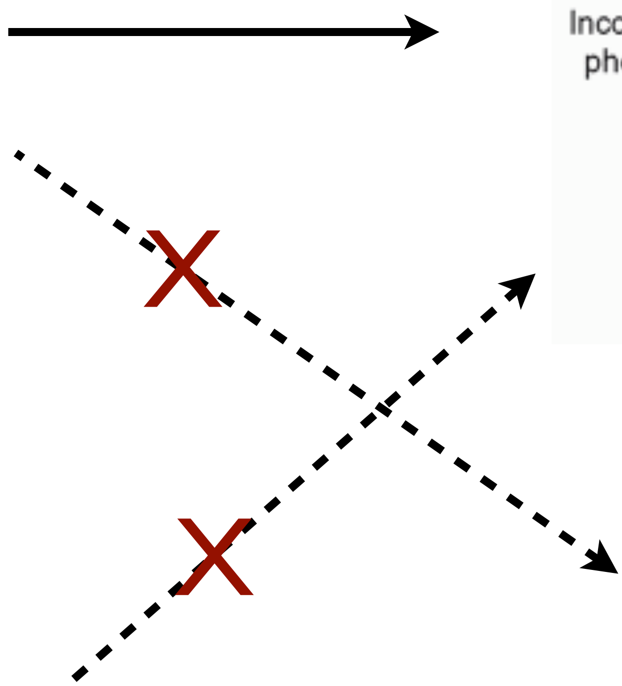
**Proposition: this approach leads to a proper understanding of neutrinos and isospin.**

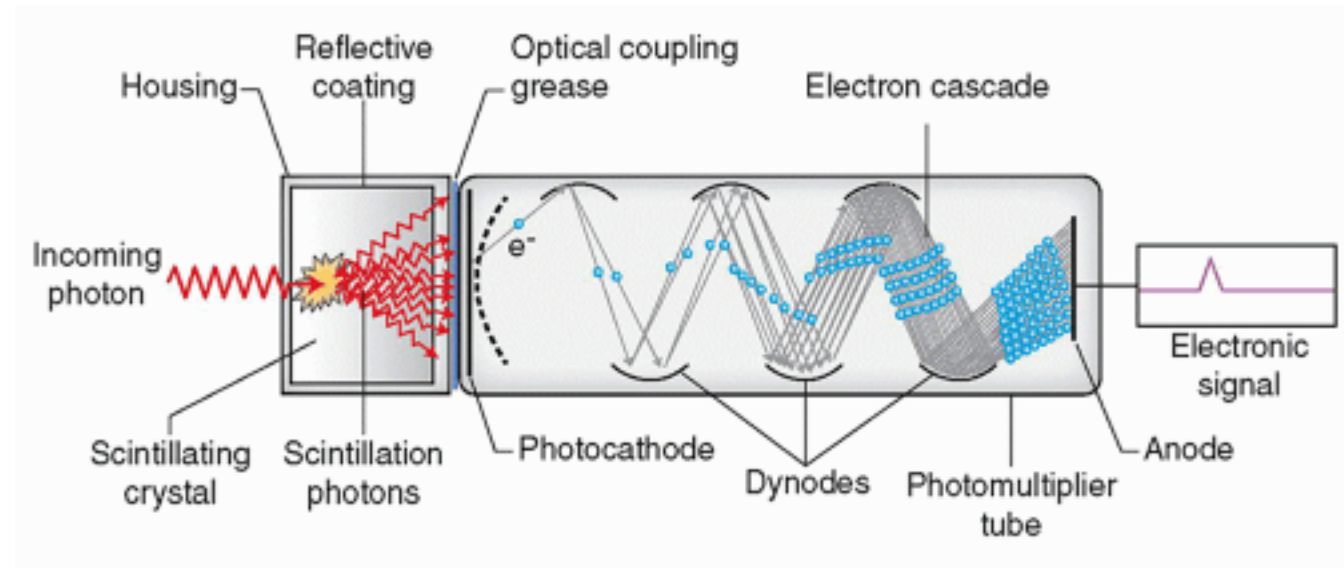
Note: we introduce here an "electromagnetic isospin" concept. It remains to be validated whether it is exactly the same as the isospin concept that was introduced by Heisenberg and Wigner in the 1930s.

# Detection of electromagnetic waves

The appropriate equipment must be used for the detection of a given electromagnetic frequency:

Gamma rays

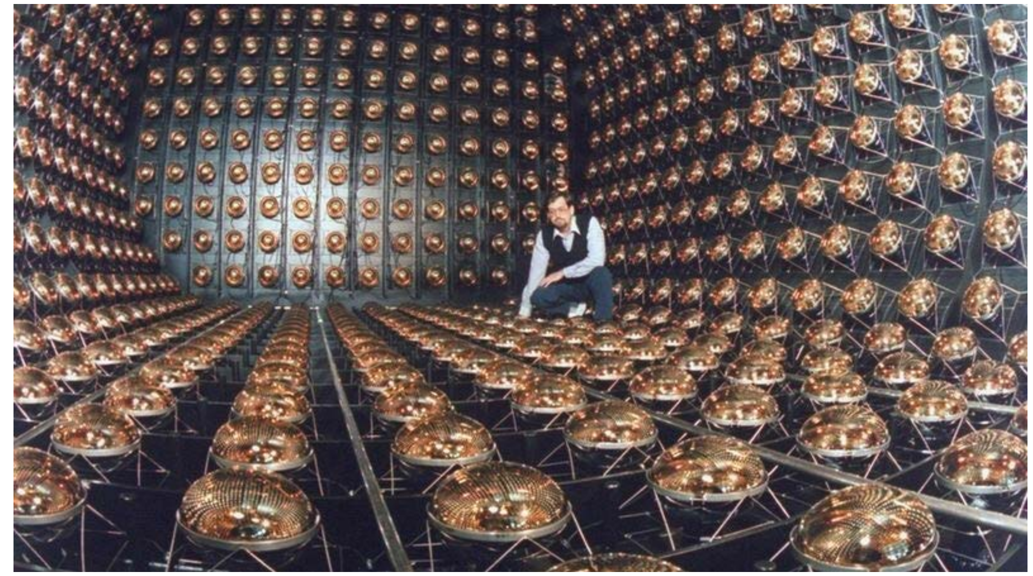




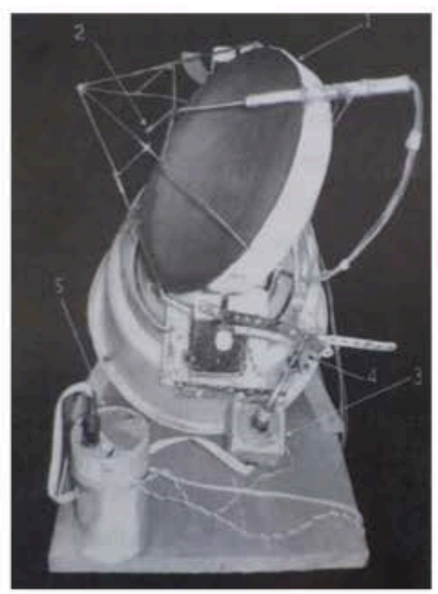

RF waves

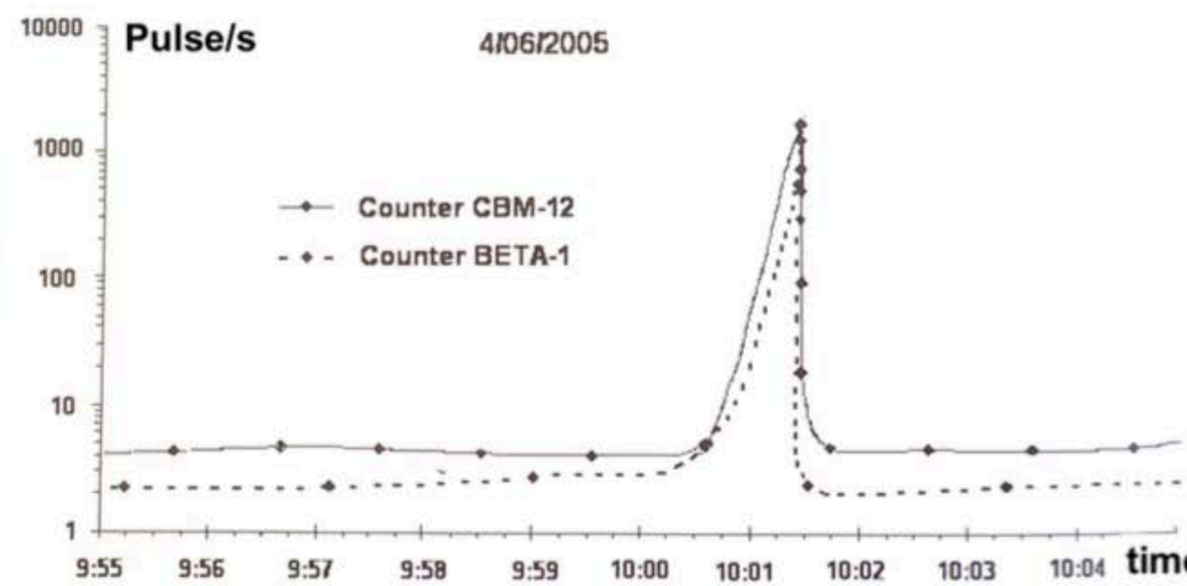
# Frequency-dependent detection of neutrino waves

High-frequency neutrinos



Low-frequency neutrinos





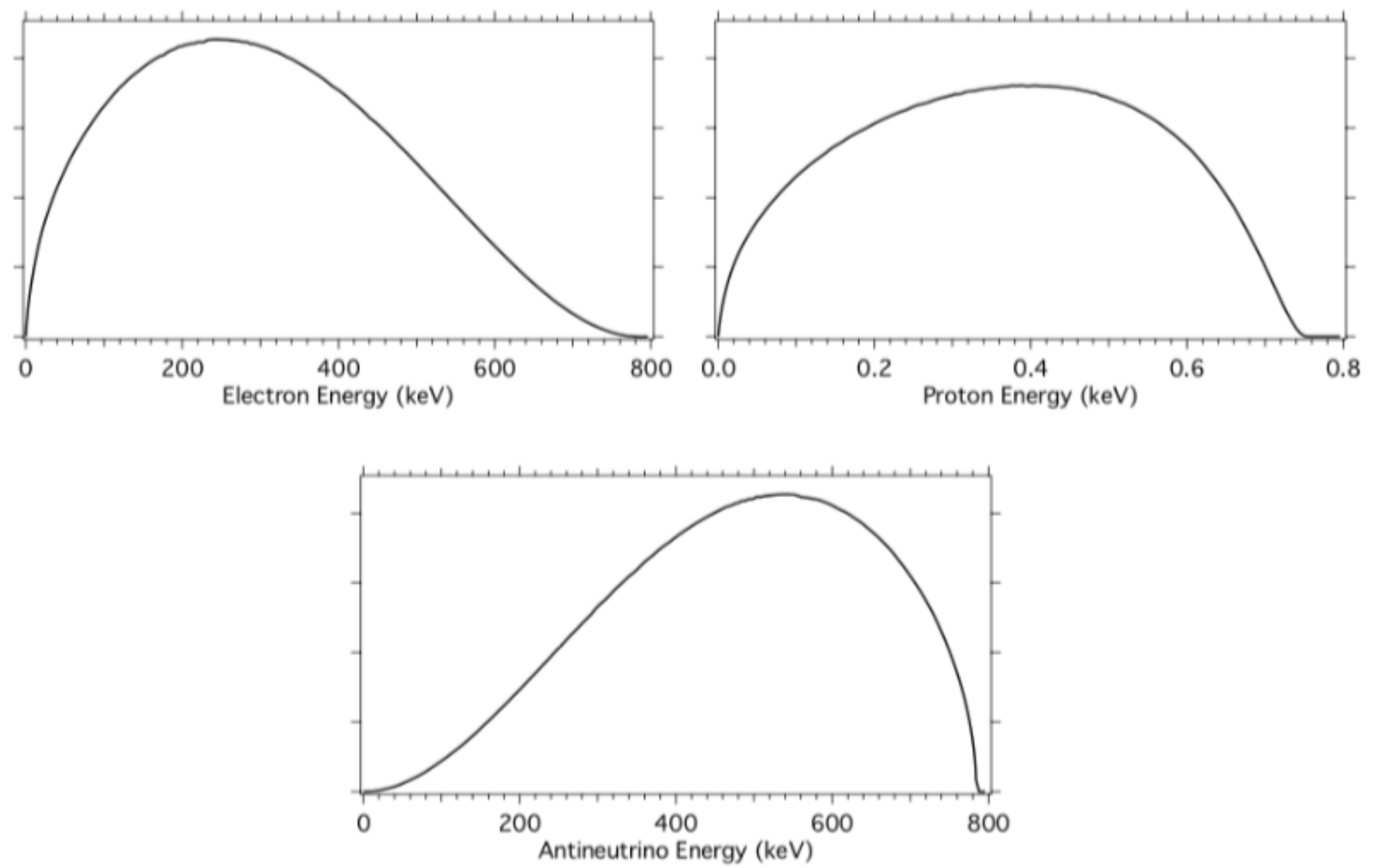

Measurement of neutrinos as electromagnetic waves. Left: parabolic metal antenna, with beta emitting $^{60}$Co material at its focal point. Right: $^{60}$Co decay rate during the experiment, as the antenna sweeps a small angle on the sky.

*A. G. Parkhomov "Bursts of count rate of beta-radioactive sources during long-term measurements", International Journal of Pure and Applied Physics, Volume 1.2 (2005), Pages 119-128*

# Neutrinos from elementary particle reactions

muon → electron + **neutrino**(s)

neutron → proton + electron + anti**neutrino**




Kinetic energy spectra for the electron, proton and antineutrino products of neutron decay

Observations:

- if neutrinos are solutions of Maxwell's equation, they are are probably a trivial solution type
- a small part of the neutrino spectrum is in the low frequency (i.e. low energy)

# Neutrinos from uranium fission

Observations:

- 6% of uranium fission energy is carried by neutrinos
- So far all neutrino measurements indicate that neutrinos travel at the speed of light
- Can we identify the light-speed traveling EMP with neutrino effect? EMP cannot be ordinary transversal wave, as it effect on materials is different from transversal wave effect.

↓

Since EMP causes electric current surges, the solution which we are looking for is a longitudinal wave

# Summary: neutrino wave properties

We are seeking electromagnetic wave solution with following properties:

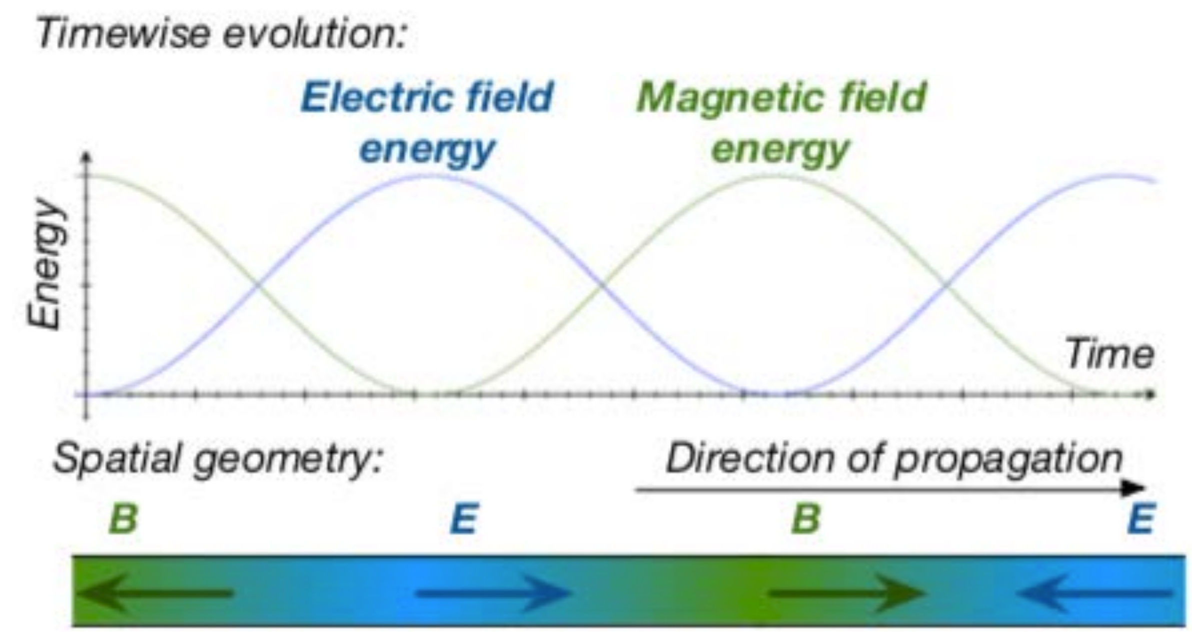
- Trivial solution of Maxwell's equation
- Travels at the speed of light
- Longitudinal wave
- Carries isospin ( $\gamma_t \rightarrow e^{I\theta}\gamma_t$ rotation)

# Neutrino wave solution

$$E_z = E_0 \sin(\omega t - kz)\,,\ B_z = B_0 \cos(\omega t - kz)$$

$$S = S_0 \cos(\omega t - kz) - iS_0 \sin(\omega t - kz)$$

where $S_0 = E_0 = B_0$ in natural units, and the wave is propagating into the $z$ direction.




Properties:

✓ Trivial solution of Maxwell's equation $\partial G = \partial^2 A_{\square} = 0$, with S≠0

✓ Travels at the speed of light

✓ Longitudinal wave: E and B fields point in the direction of propagation

✓ Carries isospin: $\gamma_t \rightarrow e^{I\theta}\gamma_t$ rotates *E* and *B* fields into each other, and rotates *S* into *iS*

# Isospin conservation requires neutrino emission

**Spin conservation example:**

Electron - positron annihilation: two circularly polarized photons are emitted.

Why photons? Spin conservation: circularly polarized photons carry away the spin of the two incoming particles.

**Isospin conservation example:**

What is the difference between an electron and a muon? Both have almost the same g-factor, which means that the muon is an energized (scaled down) version of the electron.

Its charge radius has never been measured, but our theory tells us that it is precisely

$r_0 = 2.82/206.77$ fm = 0.0136 fm

Since the muon→electron decay is always accompanied by neutrino emissions, the muon must have non-zero isospin. In contrast, the electron does not have isospin. The difference is therefore carried away by neutrinos.

**Thank you for your attention!**